

# Direct Adaptive Robust Observer based Heave Motion Predictor for Optimal Control of Wave Energy Converters

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**Abstract:** Wave Energy Converters (WEC) can achieve high energy extraction efficiency by optimal control of the PTO system. Optimal controller should achieve high tracking performance and compensate the time delay between sensors and actuators. The controller needs a predictive trajectory as the input reference. On the other hand, due to the variable sea condition and sensor noise, the heave signal measured by IMU cannot be applied to the controller directly. To extract the accurate heave motion from the signal in presence of uncertainties and sensor noise as well as compensate the time delay, a novel heave motion predictor is designed in this paper. Firstly, a heave motion dynamic proximation is established by analysis of the spectrum of IMU signal. Then the direct adaptive robust observer is applied to estimate the parameters of the heave motion dynamic proximation. Finally, the predictive trajectory will be produced by the predictive model based on the dynamic proximation. Simulation results indicate that the proposed heave motion predictor has high heave motion predict precision and robustness to uncertainties and sensor noise.

## 1. Introduction

Wave Energy Converters (WEC) are very promising equipment in offshore energy conversion [1],[2],[3]. In order to achieve high energy conversion efficiency of the system, the vertical motion of buoys excited by waves must be track precisely by the PTO system. Various control strategies have been proposed before. In [4], a Lyapunov direct method and disturbance observers based nonlinear controller was designed for a wave energy converter system. In [5], a state constrained variable structure controller based on a trajectory planning approach was proposed. In [6], a back-stepping method based nonlinear adaptive robust controller (ARC) was proposed for high tracking performance with model and parameters uncertainties.

Generally, the above mentioned WEC controllers rely on the heave motion signals measured by IMU as the reference input. The controller is designed to track the signals precisely. However, the ocean environment is complex and variable. The waves have relatively weak periodicity, so it is hard to establish the accurate dynamic model of waves. Besides, the IMU signal is inaccurate because of the sensor noise and the external disturbance. Furthermore, the time delay between the actuator and the sensor restricts the direct application of the IMU signals. Therefore, a predictive trajectory is essential to the WEC controller, which need to be planned in real time and ahead of the IMU signal.

In [7], a kind of high order sliding mode observer was proposed to estimate the state of system, but it suffered from chattering problem and is sensitive to the upper bound knowledge. In [8], an extended state observer (ESO) was designed to estimate all states and external disturbance in the ADRC framework. But the ESO relies on the integrator chain form of the system equations. In [9], a model predictive trajectory planner (MTTP) based on MPC was proposed to produce a smooth trajectory for the controller, but it did not take the time delay into consideration, and it suffered from the

exponentially increasing calculation burden. In [10], A kalman Observer based heave motion trajectory planner was proposed to extract real heave motion from the sensor noise, but Kalman observer is sensitive to the process model which can not be obtained in this scenario.

In this paper, a noval direct adaptive robust observer based heave motion predictor is proposed. The predictor will extrat the real heave motion from the noise environment and generate a predictive trajectory to compensate the time delay between actuators and sensors. A proximate heave dynamic model is established. The direct adaptive robust observer is designed based on backstepping method. The observer contains the parameter adaptation as the feedforward model compensation. Then the nonlinear robust feedback guarantees the robustness of the estimation. The observation of states is based on output feedback. In the last, simulation is conducted based on the MSS tool box of MATLAB.

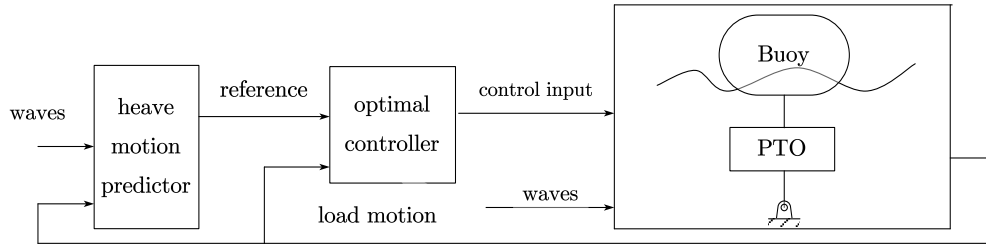


Fig. 1 Control structure of WEC system

## 2. Heave Motion Predictor

### 2.1 Heave Motion Proximation

As shown in Fig.1, the heave motion signal measured by IMU is transmitted to the predictor. Although the wave has weak periodicity, it is not totally chaotic. The measured vertical motion  $w(t)$  can be decomposed into a series of  $M$  (modes) sine waves and an bias  $p(t)$  between two time points and a period  $T$ . Therefore, the heave motion model over a period can be described as follow [10]

$$w(t) = \sum_{i=1}^M \underbrace{A_i \sin(2\pi f_i t + \varphi_i)}_{y_d(t)} + p(t), \quad i = 1, \dots, M \quad (1)$$

Where  $A_i$  is the amplitude,  $f_i$  is the frequency,  $\varphi_i$  is the phase of the  $i$ -th mode and  $p(t)$  is the uncertainties. The differential equations with uncertainties of one mode can be written as follow

$$\begin{aligned} \dot{X} &= A_i X + C_i' u + C_i' F = \begin{pmatrix} 0 & 1 \\ -(2\pi f_i)^2 & 0 \end{pmatrix} X + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u + \begin{pmatrix} 0 \\ 1 \end{pmatrix} F \\ w_i(t) &= C_i x = (1 \ 0)x \quad i = 1, \dots, M \end{aligned} \quad (2)$$

Where  $x$  is the amplitude,  $u$  the model compensation and  $F$  the unmodelled dynamics and the initial conditions:

$$X(t_0) = X_{0,i} = \begin{pmatrix} A_i \sin(\varphi_i) \\ 2\pi A_i f_i \cos(\varphi_i) \end{pmatrix}, \quad \begin{cases} x(t_0) = A_i(t_0) \sin(\varphi_i(t_0)) \\ u(t_0) = 0 \\ F(t_0) = 0 \end{cases} \quad (3)$$

**Assumption:** Uncertain dynamic model  $F$  satisfies:  $|F(t, x_1, x_2)| \leq \delta_F(x_1, x_2, t)$ .

As shown in Fig.2, the FFT algorithm is applied to identify the parametres of component modes over a certain time horizon  $T_{pred}$  and to initialize the parameters  $A_i(t_0)$ ,  $\varphi_i(t_0)$ ,  $f_i(t_0)$  of each mode. The peak detector algorithm extracts  $M$  highest frequency of the wave signal. Since we use the sine waves to approximate the ocean waves, the dynamic model is not accurate. Direct adaptive robust observers are designed to update the uncententies and reject the disturbances of the different modes online. Summating all the modes yields the observer model:

$$\dot{X} = \underbrace{\begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & A_N \end{bmatrix}}_A X_i + \begin{bmatrix} C'_1 \\ C'_2 \\ \vdots \\ C'_N \end{bmatrix} u + \begin{bmatrix} C'_1 \\ C'_2 \\ \vdots \\ C'_N \end{bmatrix} F \quad (4)$$

$$w(t) = \underbrace{[C_1 \ C_2 \ \cdots \ C_N]}_C X, \quad x(t_0) = \underbrace{[x_{1,0} \ x_{2,0} \ \cdots \ x_{N,0}]}_{x_0}^T$$

In order to eliminate the sensor noise and acquire differential signals, a second order linear tracking differentiator is designed ahead of the observer which can be written as follows

$$f = -\frac{1}{\tau_1}(v_1 - w(k)) + 4v_2, \quad v_1(k+1) = v_1(k) + h \cdot v_2, \quad v_2(k+1) = v_2(k) + h \cdot f \quad (5)$$

where  $h$  is the sampling step,  $\tau_1$  is time constant and  $v_j$  is the derivative of the measurement input  $w(t)$ .

## 2.2 Direct adaptive robust observer

The nonlinear state observers are constructed to continuously estimate the have amplitude. The proposed observers' model of one mode can be formulated as follows:

$$\ddot{x} = -(2\pi f_i)^2 x + u + F \quad (6)$$

The main objective is to design the model compensation  $u$ . As shown in Fig.3, in order to simplify the feedback design process, we choose to use a PD feedback  $Z$ , which is given by

$$\begin{cases} Z = \dot{e} + k_1 e = \dot{y} - \dot{y}_{eq} \\ \dot{y}_{eq} = \dot{y}_d - k_1 e \\ e = y - y_d \end{cases} \quad (7)$$

The error dynamic is:

$$\dot{Z} = \ddot{y} - \ddot{y}_{eq} = u + Ay + F - \ddot{y}_{eq} \quad (8)$$

Define the model compensation  $u$  as

$$\begin{aligned} u &= u_a + u_{s1} + u_{s2} \\ u_a &= \ddot{y}_d - Ay_d - \hat{F} \end{aligned} \quad (9)$$

Where  $\hat{F}$  is the estimation of uncertain dynamic, which is considered constant at each calculate step. To ensure the  $\hat{F}$  is bounded, the projection is defined as follows:

$$\text{Proj}_{\hat{F}}(\bullet) = \begin{cases} 0, & \text{if } \hat{F} \geq \delta_F \text{ and } \bullet \leq 0 \\ \bullet, & \text{else} \\ 0 & \text{if } \hat{F} \leq \delta_F \text{ and } \bullet \geq 0 \end{cases} \quad (10)$$

By using the gradient descent adaptive law given by

$$\dot{\hat{F}} = \text{Proj}_{\hat{F}}(-\gamma Z), \quad \gamma > 0 \quad (11)$$

By combining (6) and (7), the nominal stabilizing feedback can be obtained

$$u_{s1} = -k_2 - Ae - k_1 \dot{e} \quad (12)$$

where  $k_1$  and  $k_2$  are positive feedback gains.  $h(Z)$  is a nonlinear gain. So the error dynamic becomes

$$\dot{Z} + k_2 Z = u_{s2} + \tilde{F} \quad (13)$$

Where  $\tilde{F} = \hat{F} - F$  represents the effects of all model uncertainties and it is bounded with a known function  $h$ :

$$|\tilde{F} = \hat{F} - F| \leq h(Z) \quad (14)$$

With these known system characteristics, the robust feedback  $u_{s2}$  can be chosen as

$$u_{s2} = -h(Z) \text{sat}(Z) \quad (15)$$

Where  $sat(\bullet)$  denotes the continuous saturation function to obtain a continuous approximation of  $hsgn(\bullet)$ . Define a positive definite (p.d) Lyapunov function as

$$V_1 = \frac{1}{2}Z^2 + \frac{1}{2}\tilde{F}^2 \quad (16)$$

$$\dot{V}_1 = Z\dot{Z} + \tilde{F}\dot{\tilde{F}}$$

Its time derivative is

$$\begin{aligned} &= Z(-hsat(Z) + \dot{F} - k_1Z) + \tilde{F}(\dot{\tilde{F}} - \dot{F}) \\ &\leq -k_1Z^2 < 0 \end{aligned} \quad (17)$$

So the observer is closed loop stable.  $\tau_2$  is the predictive time, which can be set as 1s. The structure of the heave motion predictor is shown in Fig.2.

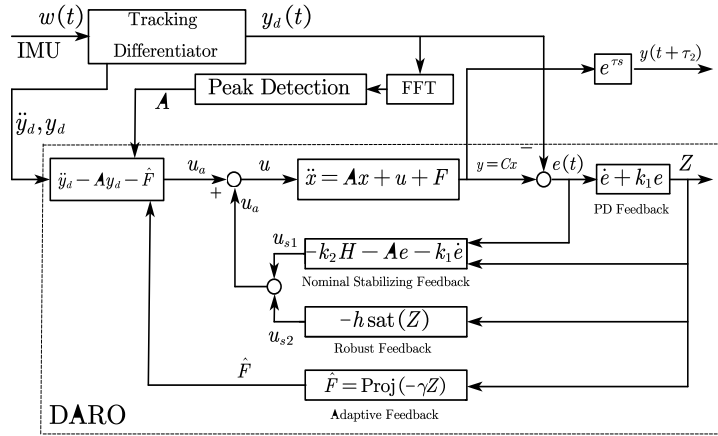
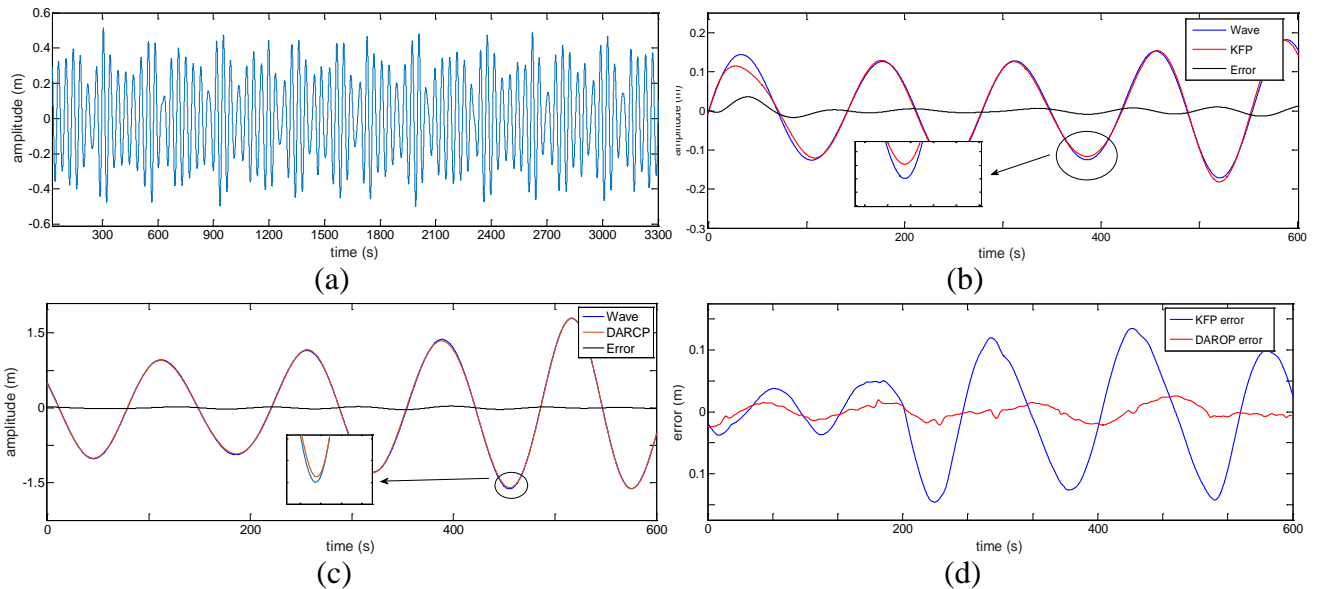


Fig. 2 Structure of the heave motion predictor

### 3. Simulation Results

The simulation is conducted based on the waves of sea state 3 generated by MSS tool box of MATLAB.

As shown in Fig.3, (a) presents the simulation waves generated by MSS tool box of MATLAB. The prediction time is set to be 1s. The reference wave shown in (b) and (c) is ahead of the original IMU signals by 1s. The proposed algorithm DAROP is compared with the Kalman filter based predictor KFP. The adaptive compensation law in DAROP deals with the model uncertainties and sensor noise, so the estimation precision is enhanced. As shown in (e), the DAROP maintains good robustness in amplitude fluctuation area, because of the nonlinear robust feedback in the algorithm.



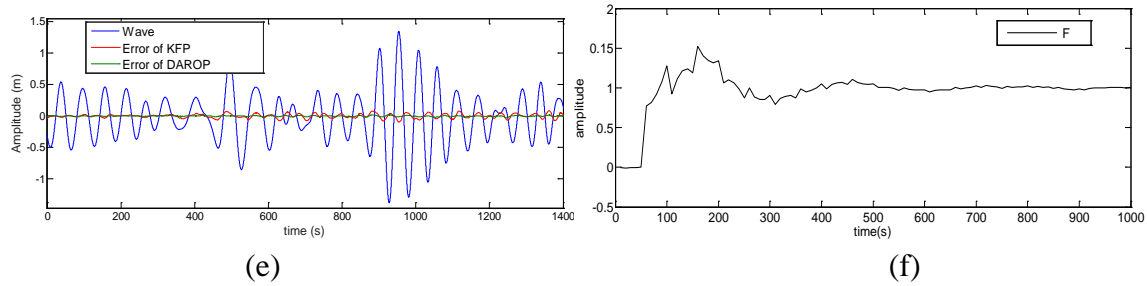


Fig. 3 (a) Simulation waves, (b) KFP prediction error, (c) DARCP prediction error, (d) error comparison of two predictor, (e) prediction error comparison in fluctuation area, (f) disturbance estimation

#### 4. Conclusion

In this paper, a predictive heave motion trajectory is proposed for the WEC system. Predictive heave motion trajectory can be generated in real time, which can be used as the reference input of the WEC controller. The algorithm contains adaptive feedforward compensation and nonlinear robust feedback. The prediction precision is enhanced compared with the previous KF based algorithm, and it shows good robustness to model uncertainties and sensor noises.

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